Using the FTC to Solve an Initial Value Problem

Sometimes when we are trying to find our original function that gave us a differential equation we find out that we don't have the methods necessary to find the antiderivative. But often times, thanks to what our calculators can do now, we can simply write an integral that would give us the antiderivative if we knew how to find it and add to that so that our initial conditions will also be met.

This is almost like magic – using the FTC wand to create a solution out of thin air! It is really quite ingenious. It solves the problem and allows to actually evaluate the original function for any value of x because our calculators are just so darned smart nowadays!

Let's look at a couple of examples.

A: Solve the initial value problem using the FTC (your solution will contain a definite integral)

$f'(x)=e^{-x^2}$ f (7) = 3	This is the derivative and initial conditions.
$f(x)=\int_7^x e^{-t^2}\,dt+3$	We have no methods to find the antiderivative of this function but we know we want to take the integral, so what we will do is write an integral that will also satisfy the initial conditions.

Look at what we did a bit more closely.
The function we are looking for is equal to the integral of our differential equation.
We set the integral up to use the FTC, with the lower limit equal to the x-value of our initial conditions.
Notice that the integrand has to contain the variable while the limit is in terms of x.
Add to the integral the y-value of the initial conditions. We do this so that when we let x = 7, we will get the correct y-value: $f(7)=\int_7^7 e^{-t^2}\,dt+3=0+3$.

And, we can also find the value of our function at any other value of x by using our calculators:
$f(-2)=\int_7^{-2} e^{-t^2}\,dt+3\approx 1.2317$. Clever!!

B: Solve the initial value problem using the FTC (your solution will contain a definite integral)

$F'(x)=e^{\cos x}$ F(2) = 9	This is the derivative and initial conditions.
$F(x)=\int_2^x e^{\cos t}\,dt+9$	Using the FTC, write an integral that will also satisfy the initial conditions.